ISSN 2348-1218 (print)
International Journal of Interdisciplinary Research and Innovations ISSN 2348-1226 (online)
Vol. 6, Issue 2, pp: (339-353), Month: April - June 2018, Available at: www.researchpublish.com

Bayesian Survival Analysis of Weibull Distribution Assuming Various Loss Structure

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Abstract: **In this paper, two parameter Weibulldistribution model is considered for Bayesian risk analysis of survival function by assuming non-informative and informative priors such as Jeffrey's, Extended Jeffrey's and Lognormal-Inverted gamma using with different type of loss functions as Squared error loss function (SELF), General entropy loss function (GELF), Quadratic loss function (QLF), Weighted loss function(WLF) and Squared logarithmic loss function(SLLF). To illustrate the methodology, simulation study is carried out and done the analysis.**

Keywords: **Bayesian analysis, Bayes Risk, General entropy loss function, Maximum likelihoodestimation, Quadratic loss function, Squared logarithmic loss function, Squared error loss function, Weibulldistribution, Weighted loss function.**

1. INTRODUCTION

The Weibull distribution is named after its originator, the Swedish physicist WallodiWeibull, who in 1939 used it to model the distribution of breaking strength of the materials and in 1951 for a wide range of other applications. The distribution has been widely studied since its inception. It is one of the best known and most applicable lifetime distribution. It adequately describes observed failures of many different types of components and phenomena. Its application, in connection with lifetimes of various manufactured items, has been widely advocated and it has been used to model a variety of life behaviors such as failures, success, incidents, occasional events, etc.Sinha (1986) has determined the Bayes estimate of reliability function and hazard rate of the Weibull distribution by using squared error loss function. Al Omari Mohammed Ahmed and et al., (2011) obtained the Bayesian survival estimator with censored data using Jeffrey's prior and extension of Jeffrey's prior information. Chris BambayGuure and et al., (2012) studied the Bayesian estimator for both scale and shape parameters of Weibull distribution using extension of Jeffrey's prior with Squared error loss, General entropy loss and Linex loss functions.Chris BambayGuure and et al., (2014) have studied the Bayes and frequentist estimators for the two parameter Weibull failure time distribution by using non-informative prior and generalization of the non-informative prior and also the reliability and hazard functions were derived under Squared error loss, General entropy loss and Linex loss functions.Lavanya, A. and Leo Alexander, T (2016) studied the problem of estimation of the survival function under the Constant Shape Bi-Weibull failure distribution by using extension of Jeffrey's prior with Squared error loss, General entropy loss and Linex loss functions.In the above all studies, the non-informative prior is assumed for estimating the parameters and also three loss functions namely, SELF, LINEX and GELF are used to carried out the analysis.Venkatesan,G and Saranya,P (2018) studied the performance of maximum likelihood estimation and Bayesian estimation of survival function of two parameter distribution assuming informative prior under various loss functions through simulation study.Till now, only estimation of the parametersare studied, no one extended the study to risk analysis of survival function of this model, we proposed to study the problem of Bayes risk analysis of survival function.

In our study, we proposed to obtain Bayes risk of survival function of the two parameter Weibull distribution using non-informative and informative priors such as Jeffrey's, Extension of Jeffrey's and Lognormal-Inverted gamma priors under Squared error loss function, General entropy loss function, Quadratic loss function, Weighted loss function and Squared logarithmic loss function.To illustrate the methodology, through simulation study is carried out and performance of the models areanalyzed.

2. MAXIMUM LIKELIHOOD ESTIMATION

Let $t_1,t_2,..,t_n$ be a random sample of size n from Weibull distribution with scale parameter (β) and shape parameter (α). The probability density function is

$$f(t;\alpha,\beta) = \frac{\alpha}{\beta}t^{\alpha-1}exp\left(\frac{-t^{\alpha}}{\beta}\right),\ t>0,\alpha>0,\beta>0 \qquad \dots (2.1)$$

The cumulative distribution function is

$$F(t;\alpha,\beta) = 1-exp\left(\frac{-t^{\alpha}}{\beta}\right),\ t>0,\alpha>0,\beta>0 \qquad \dots(2.2)$$

The survival function of t is

$$S(t;\alpha,\beta) = exp\left(\frac{-t^{\alpha}}{\beta}\right) \qquad \dots(2.3)$$

The likelihood function of t is

$$L(t;\alpha,\beta) = \prod_{i=1}^{n}\left\{\frac{\alpha}{\beta}{t_i}^{\alpha-1}exp\left(\frac{-{t_i}^{\alpha}}{\beta}\right)\right\} \qquad \dots(2.4)$$

Using the principle of MLE, we get

$$\hat{\beta} = \frac{1}{n}\sum_{i=1}^{n}{t_i}^{\hat{\alpha}} \qquad \dots(2.5)$$

where$\hat{\alpha}$ can be determined by using Newton-Raphson method and taking the initial value ofα as $\alpha_i$as per our convenience and iterating the process till it converges.

$$\alpha_{i+1} = \alpha_i - \frac{\frac{n}{\alpha}+\sum_{i=1}^{n}logt_i-\frac{\sum_{i=1}^{n}{t_i}^{\alpha}logt_i}{\frac{1}{n}\sum_{i=1}^{n}{t_i}^{\alpha}}}{-\left\{\frac{n}{\alpha^2}+\frac{\sum_{i=1}^{n}{t_i}^{\alpha}(logt_i)^2}{\frac{1}{n}\sum_{i=1}^{n}{t_i}^{\alpha}}\right\}} \qquad \dots (2.6)$$

The estimate of the survival function of the Weibull distribution under the MLE is

$$\hat{S}(t_i) = exp\left[-\left(\frac{{t_i}^{\hat{\alpha}}}{\hat{\beta}}\right)\right] \qquad \dots (2.7)$$

3. BAYESIAN ESTIMATION

Bayesian Estimation approach makes use of prior knowledge about the parameters as well as the available data.

3.1. Posterior distribution using Non-informative prior

When prior knowledge about the parameter is not available, it is possible to make use of the non-informative prior in Bayesian analysis. We use the Jeffrey's and Extension of Jeffrey's prior information, where Jeffrey's prior is the square root of the determinant of the Fisher information.

3.1.1. Jeffrey's Prior

The Jeffrey's prior of Weibull distribution is

$$v_1(\alpha,\beta) \propto \frac{1}{\alpha\beta} \qquad \dots (3.1)$$

The posterior distribution of the parameters α and β is obtained by multiplying the equation (3.1) and (2.4) as

$$\pi_1(\alpha,\beta) \propto \left(\frac{1}{\alpha\beta}\right)\prod_{i=1}^{n}\left\{\frac{\alpha}{\beta}t^{\alpha-1}exp\left(\frac{-t^{\alpha}}{\beta}\right)\right\} \qquad \dots (3.2)$$

3.1.2. Extension of Jeffrey's Prior

The Extension of Jeffrey's prior of Weibull distribution is

$$v_2(\alpha,\beta) \propto \left(\frac{1}{\alpha\beta}\right)^{2m}, m \in R^+ \quad \text{. . . (3.3)}$$

The posterior distribution of the parameters α and β is obtained by multiplying the equation (3.3) and (2.4) as

$$\pi_2(\alpha,\beta) \propto \left(\frac{1}{\alpha\beta}\right)^{2m} \prod_{i=1}^{n} \left\{\frac{\alpha}{\beta} t^{\alpha-1} exp\left(\frac{-t^{\alpha}}{\beta}\right)\right\} \quad \text{. . . (3.4)}$$

3.2. Posterior distribution using Informative prior

Bayesian estimation approach when we have knowledge on the parameters, the informative prior is preferred.We use the informative prior such as Lognormal-Inverted gamma prior information.

3.2.1. Lognormal-Inverted Gamma prior

The shape parameter α follows Lognormal distribution with hyperparameter c and scale parameter β follows Inverted-Gamma distribution with hyperparametersa and b. The joint prior distribution of α and β is

$$v_3(\alpha,\beta) \propto \frac{1}{\alpha} exp\left\{\frac{-(log\ \alpha)^2}{2c^2}\right\}\left(\frac{1}{\beta}\right)^{a+1} exp\left(\frac{-b}{\beta}\right), \quad \alpha,\beta > 0 \quad \text{...(3.5)}$$

The posterior distribution of the parameters α and β is obtained by multiplying the equation (3.5) and (2.4) as

$$\pi_3(\alpha,\beta) \propto \prod_{i=1}^{n} \left\{\frac{\alpha}{\beta} t^{\alpha-1} exp\left(\frac{-t^{\alpha}}{\beta}\right)\right\}\left(\frac{1}{\alpha}\right) exp\left\{\frac{-(log\ \alpha)^2}{2c^2}\right\}\left(\frac{1}{\beta}\right)^{a+1} exp\left(\frac{-b}{\beta}\right) \quad \text{...(3.6)}$$

4. BAYES RISK OF SURVIVAL FUNCTION UNDER DIFFERENT LOSS FUNCTION

The loss function $L(\theta,\hat{\theta})$is a measure of the error which represents the loss incurred by making an estimation when the true value of parameter θ and the estimated value $\hat{\theta}$. The Bayes risk is the posterior expected loss. The Bayes risk is defined by $R(\theta,\hat{\theta}) = E[L(\theta,\hat{\theta})]$.

4.1. Squared Error Loss Function

The squared error loss function is, $L(\theta,\hat{\theta}) = (\hat{\theta}-\theta)^2$ ...(4.1)

The squared error loss function gives equal weightage to both over and under estimation. The Bayes estimator of $\theta$is,$\hat{\theta}_{BS} = E(\theta)$. ...(4.2)

The Bayes risk is,$R(\theta,\hat{\theta}) = E(\theta^2) - [E(\theta)]^2$. ...(4.3)

4.1.1.Jeffrey's prior under squared error loss function

The Bayes risk of survival function under squared error loss function is

$$R[\hat{S}(t_i)]_{BS} = E\left\{\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^2\right\} - \left[E\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\right]^2 =$$

$$\frac{\iint \left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^2 \pi_1(\alpha,\beta)d\beta d\alpha}{\iint \pi_1(\alpha,\beta)d\beta d\alpha} - \left\{\frac{\iint exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\pi_1(\alpha,\beta)d\beta d\alpha}{\iint \pi_1(\alpha,\beta)d\beta d\alpha}\right\}^2 \quad \text{...(4.4)}$$

The above equation contains a ratio of two integrals which cannot be solved analytically, so we use Lindley's approximation procedure to estimate the survival function.Using Lindley's approximation, the expansion of

$\frac{\int u(\theta)v(\theta)[L(\theta)]d\theta}{\int v(\theta)[L(\theta)]d\theta}$can be performed as

$$\hat{\theta} = u + \frac{1}{2}[u_{11}\sigma_{11} + u_{22}\sigma_{22}] + u_1\rho_1\sigma_{11} + u_2\rho_2\sigma_{22} + \frac{1}{2}[L_{30}u_1{\sigma^2}_{11} + L_{03}u_2{\sigma^2}_{22}] \quad \text{... (4.5)}$$

where L is the log likelihood function and ρ is the logarithmic of prior distribution.

Let $u = \left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^2$ and $e = -\left(\frac{t_i^{\alpha}}{\beta}\right)$, where $\alpha = \hat{\alpha}$ and $\beta = \hat{\beta}$ is given in equation (2.6) and (2.5) respectively. It is for notational convenience.

$$u_1 = \frac{du}{d\beta} = \frac{-2ue}{\beta}\ ;\ u_{11} = \frac{du}{d\beta^2} = \frac{4ue}{\beta^2}(e+1);\ \ u_2 = \frac{du}{d\alpha} = 2ue(logt_i)\ ;\ u_{22} = \frac{du}{d\alpha^2} = 2ue(logt_i)^2(1+2e) \quad \ldots(4.6)$$

$$\rho(\alpha,\beta) = -\log\alpha - log\beta\ ;\ \rho_1 = \frac{du}{d\beta} = -\frac{1}{\beta}\ ;\quad \rho_2 = \frac{du}{d\alpha} = -\frac{1}{\alpha} \quad \ldots(4.7)$$

$$\sigma_{11} = (-L_{20})^{-1}\ ;\ \sigma_{22} = (-L_{02})^{-1}$$

$$L_{02} = \frac{d^2L}{d\alpha^2} = -\frac{n}{\alpha^2} - \frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^2\ \ ;\ L_{03} = \frac{d^3L}{d\alpha^3} = \frac{2n}{\alpha^3} - \frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^3 \quad \ldots(4.8)$$

$$L_{20} = \frac{d^2L}{d\beta^2} = \frac{n}{\beta^2} - \frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}\quad ;\ L_{30} = \frac{d^3L}{d\beta^3} = -\frac{2n}{\beta^3} + \frac{6}{\beta^4}\sum_{i=1}^{n} t_i^{\alpha} \quad \ldots(4.9)$$

Let $u = exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]$; $u_1 = \frac{du}{d\beta} = \frac{-ue}{\beta}$; $u_{11} = \frac{du}{d\beta^2} = \frac{ue}{\beta^2}(e+2)$; $u_2 = \frac{du}{d\alpha} = ue(logt_i)$;

$$u_{22} = \frac{du}{d\alpha^2} = 2ue(logt_i)^2(1+e) \quad \ldots(4.10)$$

The Bayes risk of survival function using Jeffrey's prior under squared error loss function $R\left[\hat{S}(t_i)\right]_{BS}$ is

$$\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^2 + \frac{1}{2}\left\{\frac{\frac{4ue}{\beta^2}(e+1)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{2ue(logt_i)^2(1+2e)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^2\right)}\right\} + \frac{\left(\frac{-2ue}{\beta}\right)\left(\frac{-1}{\beta}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{(2ue(logt_i))\left(\frac{-1}{\alpha}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^2\right)} + \frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n} t_i^{\alpha}\left(\frac{-2ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}\right]^2} + \frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^3\right][2ue(logt_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^2\right]^2}\right) - \left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right] + \frac{1}{2}\left\{\frac{\frac{ue}{\beta^2}(e+2)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{2ue(logt_i)^2(1+e)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^2\right)}\right\} + \frac{\left(\frac{ue}{\beta}\right)\left(\frac{-1}{\beta}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{(ue(logt_i))\left(\frac{-1}{\alpha}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^2\right)} + \frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n} t_i^{\alpha}\left(\frac{ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}\right]^2} + \frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^3\right][ue(logt_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^2\right]^2}\right)\right\}^2 \quad \ldots(4.11)$$

4.1.2.Extension of Jeffrey's prior under squared error loss function

The Bayes risk of survival function using extended Jeffrey's prior under squared error loss function is

$$R\left[\hat{S}(t_i)\right]_{BS} = E\left\{\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^2\right\} - \left[E\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\right]^2 = \frac{\iint\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^2\pi_2(\alpha,\beta)d\beta d\alpha}{\iint \pi_2(\alpha,\beta)d\beta d\alpha} - \left\{\frac{\iint exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\pi_3(\alpha,\beta)d\beta d\alpha}{\iint \pi_3(\alpha,\beta)d\beta d\alpha}\right\}^2 \ldots(4.12)$$

$$\rho(\alpha,\beta) = -\log(\alpha^{2m}) - log(\beta^{2m})\ ;\ \rho_1 = \frac{du}{d\beta} = \frac{-1}{\beta^{2m}}\ ;\quad \rho_2 = \frac{du}{d\alpha} = -\frac{1}{\alpha^{2m}} \quad \ldots(4.13)$$

The procedure of Lindley' approximation used in 4.1.1 to obtained the Bayes risk of survival function using extended Jeffrey's prior under squared error loss function $R\left[\hat{S}(t_i)\right]_{BS}$ is

$$\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^2 + \frac{1}{2}\left\{\frac{\frac{4ue}{\beta^2}(e+1)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{2ue(logt_i)^2(1+2e)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^2\right)}\right\} + \frac{\left(\frac{-2ue}{\beta}\right)\left(\frac{-1}{\beta^{2m}}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{(2ue(logt_i))\left(\frac{-1}{\alpha^{2m}}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^2\right)} + \frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n} t_i^{\alpha}\left(\frac{-2ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}\right]^2} + \frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^3\right][2ue(logt_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^2\right]^2}\right) - \left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right] + \frac{1}{2}\left\{\frac{\frac{ue}{\beta^2}(e+2)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{2ue(logt_i)^2(1+e)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^2\right)}\right\} + \frac{\left(\frac{ue}{\beta}\right)\left(\frac{-1}{\beta^{2m}}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{(ue(logt_i))\left(\frac{-1}{\alpha^{2m}}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^2\right)} + \frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n} t_i^{\alpha}\left(\frac{ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}\right]^2} + \frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^3\right][ue(logt_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(logt_i)^2\right]^2}\right)\right\}^2 \quad \ldots(4.14)$$

4.1.3.Lognormal-Inverted Gamma prior under squared error loss function

The Bayes risk of survival function using Lognormal-inverted gamma prior under squared error loss function is

ISSN 2348-1218 (print)
International Journal of Interdisciplinary Research and Innovations ISSN 2348-1226 (online)
Vol. 6, Issue 2, pp: (339-353), Month: April - June 2018, Available at: www.researchpublish.com

$$R\left[\hat{S}(t_i)\right]_{BS} = E\left\{\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^2\right\} - \left[E\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\right]^2 = \frac{\iint\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^2 \pi_3(\alpha,\beta)d\beta d\alpha}{\iint \pi_3(\alpha,\beta)d\beta d\alpha} - \left\{\frac{\iint exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\pi_3(\alpha,\beta)d\beta d\alpha}{\iint \pi_3(\alpha,\beta)d\beta d\alpha}\right\}^2 \quad \ldots(4.15)$$

$$\rho(\alpha,\beta) = -\log\alpha - \frac{(\log\alpha)^2}{2c^2} - (a+1)\log\beta - \frac{b}{\beta};\ \rho_1 = \frac{du}{d\beta} = \frac{-(a+1)}{\beta} + \frac{b}{\beta^2} \quad ; \quad \rho_2 = \frac{du}{d\alpha} = -\frac{1}{\alpha} - \frac{\log\alpha}{\alpha c^2} \ldots(4.16)$$

The procedure of Lindley' approximation used in 4.1.1 to obtained the Bayes risk of survival function using Lognormal-inverted gamma prior under squared error loss function$R\left[\hat{S}(t_i)\right]_{BS}$ is

$$\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^2 + \frac{1}{2}\left\{\frac{\frac{4ue}{\beta^2}(e+1)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{2ue(\log t_i)^2(1+2e)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right)}\right\} + \frac{\left(\frac{-2ue}{\beta}\right)\left[\frac{-(a+1)}{\beta}+\frac{b}{\beta^2}\right]}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{(2ue(\log t_i))\left[-\frac{1}{\alpha}-\frac{\log\alpha}{\alpha c^2}\right]}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right)} +$$
$$\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n} t_i^{\alpha}\left(\frac{-2ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}\right]^2} + \frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^3\right][2ue(\log t_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right]^2}\right) - \left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right] + \frac{1}{2}\left\{\frac{\frac{ue}{\beta^2}(e+2)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{2ue(\log t_i)^2(1+e)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right)}\right\} +\right.$$
$$\left.\frac{\left(\frac{ue}{\beta}\right)\left(\frac{-(a+1)}{\beta}+\frac{b}{\beta^2}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{(ue(\log t_i))\left(-\frac{1}{\alpha}-\frac{\log\alpha}{\alpha c^2}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right)} + \frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n} t_i^{\alpha}\left(\frac{ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}\right]^2} + \frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^3\right][ue(\log t_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right]^2}\right)\right\}^2 \quad \ldots(4.17)$$

4.2. Quadratic Loss Function

The Quadratic loss function is,$L(\theta,\hat{\theta}) = \left(1-\frac{\hat{\theta}}{\theta}\right)^2$ …(4.18)

The Bayes estimator of θunder Quadratic loss function is,$\hat{\theta}_{BQ} = \frac{E(\theta^{-1}/t)}{E(\theta^{-2}/t)}$. …(4.19)

The Bayes risk is,$R(\theta,\hat{\theta}) = 1 - \frac{\left[E(\theta^{-1})\right]^2}{E(\theta^{-2})}$ …(4.20)

4.2.1.Jeffrey's prior under Quadratic loss function

The Bayes risk of survival function using Jeffrey's prior under Quadratic loss function is

$$R\left[\hat{S}(t_i)\right]_{BQ} = 1 - \frac{\left\{E\left\{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}\right\}\right\}^2}{E\left\{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}\right\}} = 1 - \frac{\left\{\frac{\iint\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}\pi_1(\alpha,\beta)d\beta d\alpha}{\iint\pi_1(\alpha,\beta)d\beta d\alpha}\right\}^2}{\frac{\iint\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}\pi_1(\alpha,\beta)d\beta d\alpha}{\iint\pi_1(\alpha,\beta)d\beta d\alpha}} = 1 - \frac{I_1}{I_2} \quad \ldots(4.21)$$

Where $I_1 = \left\{\frac{\iint\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}\pi_1(\alpha,\beta)d\beta d\alpha}{\iint\pi_1(\alpha,\beta)d\beta d\alpha}\right\}^2$ .Let $u = \left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}$ and $e = -\left(\frac{t_i^{\alpha}}{\beta}\right)$

$$u_1 = \frac{du}{d\beta} = \frac{ue}{\beta} \quad ; \ u_{11} = \frac{du}{d\beta^2} = \frac{ue}{\beta^2}(e-2)\ ; \ u_2 = \frac{du}{d\alpha} = -ue(\log t_i)\ ;\ u_{22} = \frac{du}{d\alpha^2} = -ue(\log t_i)^2(1-e)$$

Where $I_2 = \frac{\iint\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}\pi_1(\alpha,\beta)d\beta d\alpha}{\iint\pi_1(\alpha,\beta)d\beta d\alpha}$. Let $u = \left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}$ and $e = -\left(\frac{t_i^{\alpha}}{\beta}\right)$

$$u_1 = \frac{du}{d\beta} = \frac{2ue}{\beta}\ ;\ u_{11} = \frac{du}{d\beta^2} = \frac{4ue}{\beta^2}(e-1);\ u_2 = \frac{du}{d\alpha} = -2ue(\log t_i)\ ;\ u_{22} = \frac{du}{d\alpha^2} = 2ue(\log t_i)^2(2e-1)$$

The procedure of Lindley' approximation used in 4.1.1 to obtained the Bayes risk of survival function using Jeffrey's prior under Quadratic loss function $R\left[\hat{S}(t_i)\right]_{BQ}$is

$$1-\frac{\left\{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}+\frac{1}{2}\left\{\frac{\left(\frac{ue}{\beta^2}(e-2)\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left[-ue(logt_i)^2(1-e)\right]}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{ue}{\beta}\right)\left(\frac{-1}{\beta}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left(-ue(logt_i)\right)\left(\frac{-1}{\alpha}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}+\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\left(\frac{-2ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right]\left[2ue(logt_i)\right]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\right\}^2}{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}+\frac{1}{2}\left\{\frac{\left(\frac{4ue}{\beta^2}(e-1)\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left[2ue(logt_i)^2(2e-1)\right]}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{2ue}{\beta}\right)\left(\frac{-1}{\beta}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left(-2ue(logt_i)\right)\left(\frac{-1}{\alpha}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}+\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\left(\frac{2ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right]\left[-2ue(logt_i)\right]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)}$$

…(4.22)

4.2.2. Extension of Jeffrey's prior under Quadratic loss function

The Bayes risk of survival function under extended Jeffrey's prior under Quadratic loss function is

$$R\left[\hat{S}(t_i)\right]_{BQ}=1-\frac{\left\{E\left\{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}\right\}\right\}^2}{E\left\{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}\right\}}=1-\frac{\left\{\frac{\iint\left\{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}\right\}\pi_2(\alpha,\beta)d\beta d\alpha}{\iint\pi_2(\alpha,\beta)d\beta d\alpha}\right\}^2}{\frac{\iint\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}\pi_2(\alpha,\beta)d\beta d\alpha}{\iint\pi_2(\alpha,\beta)d\beta d\alpha}}\qquad\text{…(4.23)}$$

The procedure of Lindley' approximation used in 4.1.1 to obtained the Bayes risk of survival function using extended Jeffrey's prior under Quadratic loss function $R\left[\hat{S}(t_i)\right]_{BQ}$ is

$$1-\frac{\left\{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}+\frac{1}{2}\left\{\frac{\left(\frac{ue}{\beta^2}(e-2)\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left[-ue(logt_i)^2(1-e)\right]}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{ue}{\beta}\right)\left(\frac{-1}{\beta 2m}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left(-ue(logt_i)\right)\left(\frac{-1}{\alpha 2m}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}+\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\left(\frac{-2ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right]\left[2ue(logt_i)\right]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\right\}^2}{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}+\frac{1}{2}\left\{\frac{\left(\frac{4ue}{\beta^2}(e-1)\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left[2ue(logt_i)^2(2e-1)\right]}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{2ue}{\beta}\right)\left(\frac{-1}{\beta 2m}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left(-2ue(logt_i)\right)\left(\frac{-1}{\alpha 2m}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}+\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\left(\frac{2ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right]\left[-2ue(logt_i)\right]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)}$$

…(4.24)

4.2.3. Lognormal-Inverted Gamma prior under Quadratic loss function

The Bayes risk of survival function using Lognormal-Inverted gamma prior under Quadratic loss function is

$$R\left[\hat{S}(t_i)\right]_{BQ}=1-\frac{\left\{E\left\{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}\right\}\right\}^2}{E\left\{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}\right\}}=1-\frac{\left\{\frac{\iint\left\{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}\right\}\pi_3(\alpha,\beta)d\beta d\alpha}{\iint\pi_3(\alpha,\beta)d\beta d\alpha}\right\}^2}{\frac{\iint\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}\pi_3(\alpha,\beta)d\beta d\alpha}{\iint\pi_3(\alpha,\beta)d\beta d\alpha}}\qquad\text{…(4.25)}$$

The procedure of Lindley' approximation used in 4.1.1 to obtained the Bayes risk of survival function using Lognormal-Inverted gamma prior under Quadratic loss function$R\left[\hat{S}(t_i)\right]_{BQ}$ is

$$1-\frac{\left\{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-1}+\frac{1}{2}\left\{\frac{\left(\frac{ue}{\beta^2}(e-2)\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left[-ue(logt_i)^2(1-e)\right]}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{ue}{\beta}\right)\left(\frac{-(a+1)}{\beta}+\frac{b}{\beta^2}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left(-ue(logt_i)\right)\left(-\frac{1}{\alpha}-\frac{log\alpha}{\alpha c^2}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\left(\frac{-2ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right]\left[2ue(logt_i)\right]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)^2}{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-2}+\frac{1}{2}\left\{\frac{\left(\frac{4ue}{\beta^2}(e-1)\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left[2ue(logt_i)^2(2e-1)\right]}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{2ue}{\beta}\right)\left(\frac{-(a+1)}{\beta}+\frac{b}{\beta^2}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left(-2ue(logt_i)\right)\left(-\frac{1}{\alpha}-\frac{log\alpha}{\alpha c^2}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}+\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\left(\frac{2ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right]\left[-2ue(logt_i)\right]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)}$$

…(4.26)

4.3. Weighted Loss Function

The weighted loss function is, $L(\hat{\theta},\theta)=\frac{(\hat{\theta}-\theta)^2}{\theta}$ ...(4.27)

The Bayes estimator under weighted loss function is, $\hat{\theta}_{BW}=[E(\theta^{-1})]^{-1}$ ...(4.28)

The Bayes risk is, $R(\theta,\hat{\theta})=E(\theta)-[E(\theta^{-1})]^{-1}$ ...(4.29)

4.3.1. Jeffrey's prior under Weighted loss function

The Bayes risk of survival function using Jeffrey's prior under Weighted loss function is

$$R[\hat{S}(t_i)]_{BW}=E\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\right\}-\left[E\left\{\left[exp\left(-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right)\right]^{-1}\right\}\right]^{-1}=\frac{\iint exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\pi_1(\alpha,\beta)d\beta d\alpha}{\iint \pi_1(\alpha,\beta)d\beta d\alpha}-\left[\frac{\iint\left[exp\left(-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right)\right]^{-1}\pi_1(\alpha,\beta)d\beta d\alpha}{\iint \pi_1(\alpha,\beta)d\beta d\alpha}\right]^{-1} \quad ...(4.30)$$

Let $u=\left[exp\left(-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right)\right]^{-1}$; $u_1=\frac{du}{d\beta}=\frac{ue}{\beta}$; $u_{11}=\frac{du}{d\beta^2}=\frac{ue}{\beta^2}(e-2)$; $u_2=\frac{du}{d\alpha}=-ue(logt_i)$;

$$u_{22}=\frac{du}{d\alpha^2}=-ue(logt_i)^2(1-e) \quad ... (4.31)$$

The procedure of Lindley' approximation used in 4.1.1 to obtained the Bayes risk of survival function using Jeffrey's prior under Weighted loss function $R[\hat{S}(t_i)]_{BW}$ is

$$\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]+\frac{1}{2}\left\{\frac{\frac{ue}{\beta^2}(e+2)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}}+\frac{2ue(logt_i)^2(1+e)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{ue}{\beta}\right)\left(\frac{-1}{\beta}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}}+\frac{(ue(logt_i))\left(\frac{-1}{\alpha}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right)}+\right.$$
$$\left.\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i{}^{\alpha}\left(\frac{ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^3\right][ue(logt_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right]^2}\right)\right\}-$$
$$\left\{\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\right\}^{-1}+\frac{1}{2}\left\{\frac{\frac{ue}{\beta^2}(e-2)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}}+\frac{\left[-ue(logt_i)^2(1-e)\right]}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{ue}{\beta}\right)\left(\frac{-1}{\beta}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}}+\frac{(-ue(logt_i))\left(\frac{-1}{\alpha}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right)}+\right.$$
$$\left.\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i{}^{\alpha}\left(\frac{ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^3\right][-ue(logt_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right]^2}\right)\right\}^{-1} \quad ... (4.32)$$

4.3.2. Extension of Jeffrey's prior under Weighted loss function

The Bayes risk of survival function using extended Jeffrey's prior under Weighted loss function is

$$R[\hat{S}(t_i)]_{BW}=E\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\right\}-\left[E\left\{\left[exp\left(-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right)\right]^{-1}\right\}\right]^{-1}=\frac{\iint exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\pi_2(\alpha,\beta)d\beta d\alpha}{\iint \pi_2(\alpha,\beta)d\beta d\alpha}-\left[\frac{\iint\left[exp\left(-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right)\right]^{-1}\pi_2(\alpha,\beta)d\beta d\alpha}{\iint \pi_2(\alpha,\beta)d\beta d\alpha}\right]^{-1} \quad ...(4.33)$$

The procedure of Lindley' approximation used in 4.1.1 to obtained the Bayes risk of survival function using extended Jeffrey's prior under Weighted loss function $R[\hat{S}(t_i)]_{BW}$ is

$$\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]+\frac{1}{2}\left\{\frac{\frac{ue}{\beta^2}(e+2)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}}+\frac{2ue(logt_i)^2(1+e)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{ue}{\beta}\right)\left(\frac{-1}{\beta 2m}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}}+\frac{(ue(logt_i))\left(\frac{-1}{\alpha 2m}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right)}+\right.$$
$$\left.\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i{}^{\alpha}\left(\frac{ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^3\right][ue(logt_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right]^2}\right)\right\}-$$

$$\left\{\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\right\}^{-1}+\frac{1}{2}\left\{\frac{\frac{ue}{\beta^2}(e-2)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}}+\frac{[-ue(logt_i)^2(1-e)]}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{ue}{\beta}\right)\left(\frac{-1}{\beta 2m}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}}+\frac{(-ue(logt_i))\left(\frac{-1}{\alpha 2m}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right)}+\right.$$
$$\left.\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i{}^{\alpha}\left(\frac{ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^3\right][-ue(logt_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right]^2}\right)\right\}^{-1} \quad \ldots(4.34)$$

4.3.3. Lognormal-Inverted Gamma prior under Weighted loss function

The Bayes risk of survival function using Lognormal-Inverted gamma prior under Weighted loss function is

$$R\left[\hat{S}(t_i)\right]_{BW}=E\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\right\}-\left[E\left\{\left[exp\left(-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right)\right]^{-1}\right\}\right]^{-1}=\frac{\iint exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\pi_3(\alpha,\beta)d\beta d\alpha}{\iint \pi_3(\alpha,\beta)d\beta d\alpha}-\left[\frac{\iint\left[exp\left(-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right)\right]^{-1}\pi_3(\alpha,\beta)d\beta d\alpha}{\iint \pi_3(\alpha,\beta)d\beta d\alpha}\right]^{-1} \quad (4.35)$$

The procedure of Lindley' approximation used in 4.1.1 to obtained the Bayes risk of survival function using Lognormal-inverted gamma prior under Weighted loss function $R\left[\hat{S}(t_i)\right]_{BW}$is

$$\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]+\frac{1}{2}\left\{\frac{\frac{ue}{\beta^2}(e+2)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}}+\frac{2ue(logt_i)^2(1+e)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{ue}{\beta}\right)\left(\frac{-(a+1)}{\beta}+\frac{b}{\beta^2}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}}+\frac{(ue(logt_i))\left(-\frac{1}{\alpha}-\frac{log\alpha}{\alpha c^2}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right)}+\right.$$
$$\left.\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i{}^{\alpha}\left(\frac{ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^3\right][ue(logt_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right]^2}\right)\right\}-$$
$$\left\{\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\right\}^{-1}+\frac{1}{2}\left\{\frac{\frac{ue}{\beta^2}(e-2)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}}+\frac{[-ue(logt_i)^2(1-e)]}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{ue}{\beta}\right)\left(\frac{-(a+1)}{\beta}+\frac{b}{\beta^2}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}}+\frac{(-ue(logt_i))\left(-\frac{1}{\alpha}-\frac{log\alpha}{\alpha c^2}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right)}+\right.$$
$$\left.\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i{}^{\alpha}\left(\frac{ue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i{}^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^3\right][-ue(logt_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i{}^{\alpha}(logt_i)^2\right]^2}\right)\right\}^{-1} \quad \ldots(4.36)$$

4.4. Squared Logarithmic Loss Function

The squared logarithmic loss function is$L\left(\theta,\hat{\theta}\right)=\left(log\hat{\theta}-log\theta\right)^2$ … (4.37)

The Bayes estimator under weighted loss function is $\hat{\theta}_{BSL}=exp[E(log\ \theta)]$ …(4.38)

The Bayes posterior risk is $R\left(\theta,\hat{\theta}\right)=E[(log\theta)^2]-[E(log\theta)]^2$ …(4.39)

4.4.1. Jeffrey's prior under Squared logarithmic loss function

The Bayes risk of survival function using Jeffrey's prior under squared logarithmic loss function is

$$R\left[\hat{S}(t_i)\right]_{BSL}=E\left\{\left\{log\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\right\}\right\}^2\right\}-\left\{E\left\{log\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\right\}\right\}\right\}^2=\frac{\iint\left\{log\left[exp\left(-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right)\right]\right\}^2\pi_1(\alpha,\beta)d\beta d\alpha}{\iint \pi_1(\alpha,\beta)d\beta d\alpha}-\left\{\frac{\iint log\left\{exp\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]\right\}\pi_1(\alpha,\beta)d\beta d\alpha}{\iint \pi_1(\alpha,\beta)d\beta d\alpha}\right\}^2 \ldots(4.40)$$

Let $u=-\left(\frac{t_i{}^{\alpha}}{\beta}\right)$; $u_1=\frac{du}{d\beta}=\frac{-u}{\beta}$; $u_{11}=\frac{du}{d\beta^2}=\frac{2u}{\beta^2}$; $u_2=\frac{du}{d\alpha}=u(logt_i)$; $u_{22}=\frac{du}{d\alpha^2}=u(logt_i)^2$

Let $u=\left[-\left(\frac{t_i{}^{\alpha}}{\beta}\right)\right]^2$; $u_1=\frac{du}{d\beta}=\frac{-2u}{\beta}$; $u_{11}=\frac{du}{d\beta^2}=\frac{6u}{\beta^2}$; $u_2=\frac{du}{d\alpha}=2u(logt_i)$; $u_{22}=\frac{du}{d\alpha^2}=4u(logt_i)^2$

The procedure of Lindley' approximation used in 4.1.1 to obtained the Bayes risk of survival function using Jeffrey's prior under squared logarithmic loss function, $R\left[\hat{S}(t_i)\right]_{BSL}$ is

ISSN 2348-1218 (print)
International Journal of Interdisciplinary Research and Innovations ISSN 2348-1226 (online)
Vol. 6, Issue 2, pp: (339-353), Month: April - June 2018, Available at: www.researchpublish.com

$$\left\{-\left(\frac{t_i^{\alpha}}{\beta}\right)\right\}^2+\frac{1}{2}\left\{\frac{\left(\frac{6u}{\beta^2}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{4u(logt_i)^2}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{-2u}{\beta}\right)\left(\frac{-1}{\beta}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left(\frac{2u(logt_i)}{\beta}\right)\left(\frac{-1}{\alpha}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}+\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\left(\frac{-2u}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right]\left[2u(logt_i)\right]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)-$$

$$\left\{\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]+\frac{1}{2}\left\{\frac{\frac{2u}{\beta^2}}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{u(logt_i)^2}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{-u}{\beta}\right)\left(\frac{-1}{\beta}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{u(logt_i)\left(\frac{-1}{\alpha}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}+\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\left(\frac{-u}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right]\left[u(logt_i)\right]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\right\}^2 \quad \ldots(4.41)$$

4.4.2. Extension of Jeffrey's prior under Squared logarithmic loss function

The Bayes risk of survival function using extended Jeffrey's prior under squared logarithmic loss function is

$$R\left[\hat{S}(t_i)\right]_{BSL}=E\left\{\left\{log\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\right\}^2\right\}-\left\{E\left\{log\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\right\}\right\}^2=$$

$$\frac{\iint\left\{log\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]\right\}^2\pi_2(\alpha,\beta)d\beta d\alpha}{\iint\pi_2(\alpha,\beta)d\beta d\alpha}-\left\{\frac{\iint log\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\pi_2(\alpha,\beta)d\beta d\alpha}{\iint\pi_2(\alpha,\beta)d\beta d\alpha}\right\}^2 \quad \ldots(4.42)$$

The procedure of Lindley' approximation used in 4.1.1 to obtained the Bayes risk of survival function using extended Jeffrey's prior under squared logarithmic loss function $R\left[\hat{S}(t_i)\right]_{BSL}$ is

$$\left\{-\left(\frac{t_i^{\alpha}}{\beta}\right)\right\}^2+\frac{1}{2}\left\{\frac{\left(\frac{6u}{\beta^2}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{4u(logt_i)^2}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{-2u}{\beta}\right)\left(\frac{-1}{\beta^{2m}}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left(\frac{2u(logt_i)}{\beta}\right)\left(\frac{-1}{\alpha^{2m}}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}+\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\left(\frac{-2u}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right]\left[2u(logt_i)\right]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)-$$

$$\left\{\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]+\frac{1}{2}\left\{\frac{\frac{2u}{\beta^2}}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{u(logt_i)^2}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{-u}{\beta}\right)\left(\frac{-1}{\beta^{2m}}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{u(logt_i)\left(\frac{-1}{\alpha^{2m}}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}+\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\left(\frac{-u}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right]\left[u(logt_i)\right]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\right\}^2 \quad \ldots(4.43)$$

4.4.3. Lognormal-Inverted Gamma prior under Squared logarithmic loss function

The Bayes risk of survival function using Lognormal-Inverted gamma prior under squared logarithmic loss function is

$$R\left[\hat{S}(t_i)\right]_{BSL}=E\left\{\left\{log\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\right\}^2\right\}-\left\{E\left\{log\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\right\}\right\}^2=$$

$$\frac{\iint\left\{log\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]\right\}^2\pi_3(\alpha,\beta)d\beta d\alpha}{\iint\pi_3(\alpha,\beta)d\beta d\alpha}-\left\{\frac{\iint log\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\pi_3(\alpha,\beta)d\beta d\alpha}{\iint\pi_3(\alpha,\beta)d\beta d\alpha}\right\}^2 \quad \ldots(4.44)$$

The procedure of Lindley' approximation used in 4.1.1 to obtained the Bayes risk of survival function using Lognormal-inverted gamma prior under squared logarithmic loss function$R\left[\hat{S}(t_i)\right]_{BSL}$ is

$$\left\{-\left(\frac{t_i^{\alpha}}{\beta}\right)\right\}^2+\frac{1}{2}\left\{\frac{\left(\frac{6u}{\beta^2}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{4u(logt_i)^2}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{-2u}{\beta}\right)\left(\frac{-(a+1)}{\beta}+\frac{b}{\beta^2}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{\left(\frac{2u(logt_i)}{\beta}\right)\left(-\frac{1}{\alpha}-\frac{log\alpha}{\alpha c^2}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}+\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\left(\frac{-2u}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right][2u(logt_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)-$$

$$\left\{\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]+\frac{1}{2}\left\{\frac{\frac{2u}{\beta^2}}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{u(logt_i)^2}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{-u}{\beta}\right)\left(\frac{-(a+1)}{\beta}+\frac{b}{\beta^2}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{u(logt_i)\left(-\frac{1}{\alpha}-\frac{log\alpha}{\alpha c^2}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}+\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\left(\frac{-u}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right][u(logt_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\right\}^2 \quad \ldots(4.45)$$

4.5. General Entropy Loss Function

The General Entropy (GE) Loss is$L(\hat{\theta}-\theta)\propto\left(\frac{\hat{\theta}}{\theta}\right)^k-k\,ln\left(\frac{\hat{\theta}}{\theta}\right)-1$ …(4.46)

The Bayes estimator $\hat{\theta}_{BG}$of θ under General entropy loss function is$\hat{\theta}_{BG}=[E_{\theta}(\theta^{-k})]^{-\frac{1}{k}}$ …(4.47)

The Bayes posterior risk is $R(\theta,\hat{\theta})=kE[log\,\theta]-log[E(\theta^{-k})]$ …(4.48)

4.5.1. Jeffrey's prior under General entropy loss function

The Bayes risk of survival function using Jeffrey's prior under General entropy loss function is

$$R[\hat{S}(t_i)]_{BG}=kE\left\{log\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\right\}-logE\left\{\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-k}\right\}=$$

$$k\left[\frac{\iint log\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\pi_1(\alpha,\beta)d\beta d\alpha}{\iint\pi_1(\alpha,\beta)d\beta d\alpha}\right]-log\left\{\frac{\iint\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^{-k}\pi_1(\alpha,\beta)d\beta d\alpha}{\iint\pi_1(\alpha,\beta)d\beta d\alpha}\right\} \quad \ldots(4.49)$$

Let $u=\left[exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-k}$; $u_1=\frac{du}{d\beta}=\frac{kue}{\beta}$; $u_{11}=\frac{du}{d\beta^2}=\frac{kue}{\beta^2}(ke-2)$; $u_2=\frac{du}{d\alpha}=-kue(logt_i)$;

$$u_{22}=\frac{du}{d\alpha^2}=kue(logt_i)^2(ke-1)$$

The procedure of Lindley' approximation used in 4.1.1 to obtained the Bayes risk of survival function using Jeffrey's prior under General entropy loss function $R[\hat{S}(t_i)]_{BG}$is

$$k\left\{log\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}+\frac{1}{2}\left\{\frac{\frac{2u}{\beta^2}}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{u(logt_i)^2}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{-u}{\beta}\right)\left(\frac{-1}{\beta}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{u(logt_i)\left(\frac{-1}{\alpha}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}+\frac{1}{2}\left(\frac{\left[-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\right]\left(\frac{-u}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right][u(logt_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\right\}-$$

$$log\left\{\left\{exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^{-k}+\frac{1}{2}\left\{\frac{\frac{kue}{\beta^2}(ke-2)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{kue(logt_i)^2(ke-1)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}\right\}+\frac{\left(\frac{kue}{\beta}\right)\left(\frac{-1}{\beta}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}}+\frac{(-kue(logt_i))\left(\frac{-1}{\alpha}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right)}+\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n}t_i^{\alpha}\left(\frac{kue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n}t_i^{\alpha}\right]^2}+\frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^3\right][-kue(logt_i)]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n}t_i^{\alpha}(logt_i)^2\right]^2}\right)\right\} \quad \ldots(4.50)$$

4.5.2. Extension of Jeffrey's prior under General entropy loss function

The Bayes risk of survival function using extended Jeffrey's prior under General entropy loss function is

$$R\left[\hat{S}(t_i)\right]_{BG} = kE\left\{\log\left\{\exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\right\} - \log E\left\{\left[\exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-k}\right\} =$$

$$k\left[\frac{\iint \log\left\{\exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\pi_2(\alpha,\beta)\,d\beta\, d\alpha}{\iint \pi_2(\alpha,\beta)\,d\beta\, d\alpha}\right] - \log\left\{\frac{\iint \left\{\exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^{-k}\pi_2(\alpha,\beta)\,d\beta\, d\alpha}{\iint \pi_2(\alpha,\beta)\,d\beta\, d\alpha}\right\} \qquad \ldots(4.51)$$

The procedure of Lindley' approximation used in 4.1.1 to obtained the Bayes risk of survival function using extended Jeffrey's prior under General entropy loss function$R\left[\hat{S}(t_i)\right]_{BG}$ is

$$k\left\{\log\left\{\exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\} + \frac{1}{2}\left\{\frac{\frac{2u}{\beta^2}}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{u(\log t_i)^2}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right)}\right\} + \frac{\left(\frac{-u}{\beta}\right)\left(\frac{-1}{\beta^2 m}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{u(\log t_i)\left(\frac{-1}{\alpha^2 m}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right)} +\right.$$

$$\left.\frac{1}{2}\left(\frac{\left[-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n} t_i^{\alpha}\right]\left(\frac{-u}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}\right]^2} + \frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^3\right]\left[u(\log t_i)\right]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right]^2}\right)\right\} -$$

$$\log\left\{\left\{\exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^{-k} + \frac{1}{2}\left\{\frac{\frac{kue}{\beta^2}(ke-2)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{kue(\log t_i)^2(ke-1)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right)}\right\} + \frac{\left(\frac{kue}{\beta}\right)\left(\frac{-1}{\beta^2 m}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{(-kue(\log t_i))\left(\frac{-1}{\alpha^2 m}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right)} +\right.$$

$$\left.\frac{1}{2}\left(\frac{-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n} t_i^{\alpha}\left(\frac{kue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}\right]^2} + \frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^3\right]\left[-kue(\log t_i)\right]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right]^2}\right)\right\} \qquad \ldots(4.52)$$

4.5.3. Lognormal-Inverted Gamma prior under General entropy loss function

The Bayes risk of survival function using Lognormal-inverted gamma prior under General entropy loss function is

$$R\left[\hat{S}(t_i)\right]_{BG} = kE\left\{\log\left\{\exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\right\} - \log E\left\{\left[\exp\left(-\left(\frac{t_i^{\alpha}}{\beta}\right)\right)\right]^{-k}\right\} =$$

$$k\left[\frac{\iint \log\left\{\exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}\pi_2(\alpha,\beta)\,d\beta\, d\alpha}{\iint \pi_2(\alpha,\beta)\,d\beta\, d\alpha}\right] - \log\left\{\frac{\iint \left\{\exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^{-k}\pi_2(\alpha,\beta)\,d\beta\, d\alpha}{\iint \pi_2(\alpha,\beta)\,d\beta\, d\alpha}\right\} \qquad \ldots(4.53)$$

The procedure of Lindley' approximation used in 4.1.1 to obtained the Bayes risk of survival function using extended Jeffrey's prior under General entropy loss function $R\left[\hat{S}(t_i)\right]_{BG}$is

$$k\left\{\log\left\{\exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\} + \frac{1}{2}\left\{\frac{\frac{2u}{\beta^2}}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{u(\log t_i)^2}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right)}\right\} + \frac{\left(\frac{-u}{\beta}\right)\left(\frac{-(a+1)}{\beta}+\frac{b}{\beta^2}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{u(\log t_i)\left(-\frac{1}{\alpha}-\frac{\log\alpha}{\alpha c^2}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right)} +\right.$$

$$\left.\frac{1}{2}\left(\frac{\left[-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n} t_i^{\alpha}\right]\left(\frac{-u}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}\right]^2} + \frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^3\right]\left[u(\log t_i)\right]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right]^2}\right)\right\} -$$

$$\log\left\{\left\{\exp\left[-\left(\frac{t_i^{\alpha}}{\beta}\right)\right]\right\}^{-k} + \frac{1}{2}\left\{\frac{\frac{kue}{\beta^2}(ke-2)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{kue(\log t_i)^2(ke-1)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right)}\right\} + \frac{\left(\frac{kue}{\beta}\right)\left(\frac{-(a+1)}{\beta}+\frac{b}{\beta^2}\right)}{\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}} + \frac{(-kue(\log t_i))\left(-\frac{1}{\alpha}-\frac{\log\alpha}{\alpha c^2}\right)}{\left(-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right)} +\right.$$

$$\left.\frac{1}{2}\left(\frac{\left(-\frac{2n}{\beta^3}+\frac{6}{\beta^4}\sum_{i=1}^{n} t_i^{\alpha}\right)\left(\frac{kue}{\beta}\right)}{\left[\frac{n}{\beta^2}-\frac{1}{\beta^3}\sum_{i=1}^{n} t_i^{\alpha}\right]^2} + \frac{\left[\frac{2n}{\alpha^3}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^3\right]\left[-kue(\log t_i)\right]}{\left[-\frac{n}{\alpha^2}-\frac{1}{\beta}\sum_{i=1}^{n} t_i^{\alpha}(\log t_i)^2\right]^2}\right)\right\} \qquad \ldots(4.54)$$

5. SIMULATION STUDY

In this study, we chose a sample size of n=25, 50 and 100 to represent small, medium and large dataset. The Bayes risk of survival function is estimated for Weibull distribution using non-informative and informative prior under different loss functions. The values of the parameters chosen as α=0.8, 1.2, 3 and β=0.5, 1.5, 5. The values of Jeffrey,s extension are m=0.4,1.4 and for the informative priors are chosen as a=0.4,1.4; b=0.6,1.6 and c=0.9, 1.9. The values for the loss parameter is k=±0.6 and ±1.6. The results of the simulation study are discussed as follows:

The Bayes risk of survival function for two parameter Weibull distribution using non-informative prior (Jeffrey's) under various loss functions is obtained and presented in Table-1.

Table-1: Estimation of Bayes risk of Survival Function under Jeffrey's prior.

n	β	α	$R(\hat{S}(t_i))_{BS}$	$R(\hat{S}(t_i))_{BG}$				$R(\hat{S}(t_i))_{BQ}$	$R(\hat{S}(t_i))_{BW}$	$R(\hat{S}(t_i))_{BSL}$
				k=0.6	k=-0.6	k=1.6	k=-1.6			
25	0.5	0.8	4.265182e-05	1.037339e-04	1.041838e-04	7.155784e-04	7.231783e-04	5.223156e-04	9.957379e-05	5.806347e-04
		1.2	4.257094e-05	1.039427e-04	1.043913e-04	7.170099e-04	7.246235e-04	5.233461e-04	9.938550e-05	5.818016e-04
		3	4.252574e-05	1.042913e-04	1.047448e-04	7.192162e-04	7.268862e-04	5.246681e-04	9.926454e-05	5.838017e-04
	1.5	0.8	2.107351e-05	1.917451e-04	1.911227e-04	1.291328e-03	1.281395e-03	8.943163e-04	9.119160e-05	1.076715e-03
		1.2	2.102789e-05	1.922872e-04	1.916639e-04	1.294955e-03	1.285002e-03	8.966696e-04	9.104586e-05	1.079755e-03
		3	2.088850e-05	1.935041e-04	1.928630e-04	1.301875e-03	1.291728e-03	8.996960e-04	9.058339e-05	1.086860e-03
	5	0.8	2.576148e-05	1.875878e-04	1.867206e-04	1.260606e-03	1.248831e-03	8.717059e-04	1.089451e-04	1.053602e-03
		1.2	2.576776e-05	1.880556e-04	1.871867e-04	1.263397e-03	1.251624e-03	8.728944e-04	1.088466e-04	1.056272e-03
		3	2.592740e-05	1.888503e-04	1.879872e-04	1.270401e-03	1.258642e-03	8.776844e-04	1.089453e-04	1.060198e-03
50	0.5	0.8	2.166338e-05	5.345731e-05	5.359302e-05	3.720905e-04	3.743944e-04	2.767073e-04	4.998834e-05	2.984826e-04
		1.2	2.163685e-05	5.359561e-05	5.373066e-05	3.730237e-04	3.753412e-04	2.773633e-04	4.992467e-05	2.992505e-04
		3	2.161527e-05	5.367194e-05	5.380852e-05	3.735569e-04	3.758749e-04	2.777498e-04	4.987602e-05	2.996786e-04
	1.5	0.8	1.031805e-05	9.806281e-05	9.783568e-05	6.739697e-04	6.701996e-04	4.857900e-04	4.493309e-05	5.481254e-04
		1.2	1.024908e-05	9.823841e-05	9.799907e-05	6.744581e-04	6.705156e-04	4.854574e-04	4.466954e-05	5.492668e-04
		3	1.021963e-05	9.844511e-05	9.820039e-05	6.755884e-04	6.715656e-04	4.860320e-04	4.455782e-05	5.504914e-04
	5	0.8	1.259977e-05	9.500467e-05	9.479719e-05	6.547693e-04	6.515350e-04	4.744499e-04	5.314838e-05	5.306775e-04
		1.2	1.259680e-05	9.499775e-05	9.479035e-05	6.547187e-04	6.514846e-04	4.744125e-04	5.314561e-05	5.306373e-04
		3	1.251463e-05	9.482269e-05	9.461164e-05	6.534909e-04	6.502412e-04	4.737560e-04	5.281694e-05	5.296850e-04
100	0.5	0.8	1.085669e-05	2.837459e-05	2.845147e-05	1.966098e-04	1.977046e-04	1.456349e-04	2.495208e-05	1.586907e-04
		1.2	1.084541e-05	2.849843e-05	2.857918e-05	1.972910e-04	1.984143e-04	1.459563e-04	2.492548e-05	1.594465e-04
		3	1.083876e-05	2.854009e-05	2.862039e-05	1.975736e-04	1.986947e-04	1.461561e-04	2.490679e-05	1.596724e-04
	1.5	0.8	5.165557e-06	5.110406e-05	5.105948e-05	3.503831e-04	3.494704e-04	2.539399e-04	2.255066e-05	2.863242e-04
		1.2	5.131053e-06	5.129492e-05	5.124413e-05	3.513655e-04	3.503801e-04	2.541662e-04	2.242547e-05	2.874261e-04
		3	5.099897e-06	5.163554e-05	5.158347e-05	3.528279e-04	3.518097e-04	2.543505e-04	2.230392e-05	2.895904e-04
	5	0.8	6.423358e-06	4.878581e-05	4.873219e-05	3.374251e-04	3.365314e-04	2.477100e-04	2.683612e-05	2.725275e-04
		1.2	6.423776e-06	4.878674e-05	4.873358e-05	3.406515e-04	3.365428e-04	2.477165e-04	2.683790e-05	2.725341e-04
		3	6.359124e-06	4.876065e-05	4.870413e-05	3.370788e-04	3.361648e-04	2.471607e-04	2.661076e-05	2.724026e-04

From the table -1, it is observed that the Bayes risk of survival function is minimum under the SELF, WLF when $\beta<1$ and $\alpha>1$ than $\beta<1$ and $\alpha<1$ for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function is maximum under the GELF, QLF, SLLF when $\beta<1$ and $\alpha>1$ than $\beta<1$ and $\alpha<1$ for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function for Weibull distribution using non-informative prior (Jeffrey's) under SELF is better than using other loss functions in this study.

The Bayes risk of survival function for two parameter Weibull distribution using non-informative prior (Extension of Jeffrey's prior m=1.4) under various loss functions is obtained and presented in Table-2.

Table-2: Estimation of Bayes risk of Survival Function under Extension of Jeffrey's prior m=0.4.

n	β	α	$R(\hat{S}(t_i))_{BS}$	$R(\hat{S}(t_i))_{BG}$				$R(\hat{S}(t_i))_{BQ}$	$R(\hat{S}(t_i))_{BW}$	$R(\hat{S}(t_i))_{BSL}$
				k=0.6	k=-0.6	k=1.6	k=-1.6			
25	0.5	0.8	4.264769e-05	1.038768e-04	1.040891e-04	7.175574e-04	7.215405e-04	5.243452e-04	9.976163e-05	5.807725e-04
		1.2	4.255857e-05	1.040995e-04	1.042699e-04	7.192420e-04	7.226405e-04	5.256072e-04	9.952153e-05	5.819075e-04
		3	4.248356e-05	1.044814e-04	1.089581e-04	7.220780e-04	7.239239e-04	5.275223e-04	9.925440e-05	5.837600e-04
	1.5	0.8	2.109839e-05	1.915217e-04	1.913672e-04	1.288330e-03	1.284539e-03	8.916160e-04	9.107395e-05	1.076776e-03
		1.2	2.104127e-05	1.922093e-04	1.917476e-04	1.293959e-03	1.286041e-03	8.958021e-04	9.098138e-05	1.079772e-03
		3	2.087050e-05	1.937782e-04	1.925045e-04	1.305972e-03	1.287192e-03	9.035532e-04	9.064720e-05	1.086605e-03
	5	0.8	2.588012e-05	1.868456e-04	1.875005e-04	1.250909e-03	1.258756e-03	8.631970e-04	1.084476e-04	1.053713e-03
		1.2	2.587827e-05	1.874133e-04	1.878778e-04	1.254943e-03	1.260336e-03	8.654656e-04	1.084008e-04	1.056419e-03
		3	2.601578e-05	1.884523e-04	1.884376e-04	1.265052e-03	1.264241e-03	8.729035e-04	1.086273e-04	1.060368e-03
50	0.5	0.8	2.166350e-05	5.349411e-05	5.356971e-05	3.726110e-04	3.739642e-04	2.772730e-04	5.003108e-05	2.985163e-04
		1.2	2.163392e-05	5.363880e-05	5.369561e-05	3.736770e-04	3.747447e-04	2.780675e-04	4.995393e-05	2.992750e-04
		3	2.160218e-05	5.373223e-05	5.373904e-05	3.745295e-04	3.748425e-04	2.787971e-04	4.986379e-05	2.996554e-04
	1.5	0.8	1.032613e-05	9.799436e-05	9.791408e-05	6.729384e-04	6.712925e-04	4.847406e-04	4.489468e-05	5.481480e-04
		1.2	1.025458e-05	9.820536e-05	9.803590e-05	6.739882e-04	6.710129e-04	4.850120e-04	4.464512e-05	5.492797e-04
		3	1.021765e-05	9.850704e-05	9.811416e-05	6.766387e-04	6.703798e-04	4.871740e-04	4.456829e-05	5.504229e-04
	5	0.8	1.263298e-05	9.480491e-05	9.500213e-05	6.676287e-04	6.545653e-04	4.713891e-04	5.300010e-05	5.306892e-04
		1.2	1.262832e-05	9.482414e-05	9.497361e-05	6.520926e-04	6.541668e-04	4.717256e-04	5.301138e-05	5.306596e-04
		3	1.254084e-05	9.470066e-05	9.474444e-05	6.516857e-04	6.521165e-04	4.719635e-04	5.271576e-05	5.297177e-04
100	0.5	0.8	1.085660e-05	2.838829e-05	2.844165e-05	1.968164e-04	1.975193e-04	1.458640e-04	2.496397e-05	1.586988e-04
		1.2	1.084466e-05	2.851834e-05	2.856205e-05	1.975783e-04	1.981451e-04	1.462591e-04	2.493325e-05	1.594544e-04
		3	1.083507e-05	2.857376e-05	2.858614e-05	1.980555e-04	1.981995e-04	1.466439e-04	2.490482e-05	1.596712e-04
	1.5	0.8	5.167199e-06	5.108099e-05	5.108549e-05	3.500467e-04	3.498241e-04	2.535971e-04	2.254189e-05	2.863292e-04
		1.2	5.131805e-06	5.128799e-05	5.125256e-05	3.512470e-04	3.505014e-04	2.540422e-04	2.242070e-05	2.874296e-04
		3	5.099203e-06	5.166982e-05	5.154276e-05	3.533133e-04	3.512898e-04	2.548161e-04	2.230741e-05	2.895702e-04
	5	0.8	6.431061e-06	4.871885e-05	4.879925e-05	3.364161e-04	3.375522e-04	2.466415e-04	2.679939e-05	2.725311e-04
		1.2	6.430977e-06	4.872913e-05	4.879268e-05	3.365544e-04	3.374386e-04	2.467815e-04	2.680521e-05	2.725420e-04
		3	6.365176e-06	4.872100e-05	4.874614e-05	3.364869e-04	3.367787e-04	2.465324e-04	2.658654e-05	2.724137e-04

From the table-2, it is observed that the Bayes risk of survival function is minimum under the SELF, WLF when β<1 and α>1, β>1 and α>1 than β<1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. TheBayes risk of survival function is maximum under the GELF, QLF, SLLF when β<1 and α>1 than β<1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function for Weibull distribution using non-informative prior (Extension of Jeffrey's) under SELF is better than using other loss functions proposed in this study.

The Bayes risk of survival function for two parameter Weibull distribution using non-informative prior (Extension of Jeffrey's) under various loss functions is obtained and presented in Table-3.

Table-3: Estimation of Bayes risk of Survival Function under Extension of Jeffrey's prior m=1.4 .

N	B	A	$R(\hat{S}(t_i))_{BS}$	$R(\hat{S}(t_i))_{BG}$				$R(\hat{S}(t_i))_{BQ}$	$R(\hat{S}(t_i))_{BW}$	$R(\hat{S}(t_i))_{BSL}$
				k=0.6	k=-0.6	k=1.6	k=-1.6			
25	0.5	0.8	4.114546e-05	9.636825e-05	1.011213e-04	6.521940e-04	7.283729e-04	4.699152e-04	9.200134e-05	5.500866e-04
		1.2	4.173664e-05	9.652611e-05	1.016669e-04	6.524053e-04	7.349826e-04	4.698455e-04	9.334619e-05	5.179184e-04
		3	4.203288e-05	9.656671e-05	1.020394e-04	6.519652e-04	7.399892e-04	4.693126e-04	9.410449e-05	5.528210e-04
	1.5	0.8	2.079938e-05	1.929770e-04	1.885675e-04	1.313484e-03	1.253289e-03	9.154682e-04	9.174116e-05	1.072574e-03
		1.2	2.091616e-05	1.925795e-04	1.911843e-04	1.299286e-03	1.279747e-03	9.004165e-04	9.129106e-05	1.079184e-03
		3	2.086144e-05	1.927087e-04	1.935452e-04	1.291329e-03	1.301839e-03	8.900383e-04	9.046954e-05	1.086492e-03
	5	0.8	2.517330e-05	1.888902e-04	1.832784e-04	1.286846e-03	1.213369e-03	8.969444e-04	1.096344e-04	1.046772e-03
		1.2	2.536323e-05	1.889418e-04	1.852154e-04	1.279357e-03	1.230861e-03	8.878396e-04	1.093765e-04	1.052827e-03
		3	2.564451e-05	1.892769e-04	1.869993e-04	1.277993e-03	1.230861e-03	8.847265e-04	1.092865e-04	1.058437e-03
50	0.5	0.8	2.141710e-05	5.201661e-05	5.325025e-05	3.578425e-04	3.778904e-04	2.634201e-04	4.854447e-05	2.933385e-04
		1.2	2.151872e-05	5.205328e-05	5.345474e-05	3.575318e-04	3.802975e-04	2.628858e-04	4.879218e-05	2.939675e-04
		3	2.156181e-05	5.201864e-05	5.353955e-05	3.569310e-04	3.816339e-04	2.622694e-04	4.892183e-05	2.940726e-04
	1.5	0.8	1.024406e-05	9.838925e-05	9.715353e-05	6.805084e-04	6.617579e-04	4.930071e-04	4.508128e-05	5.470668e-04
		1.2	1.020906e-05	9.833309e-05	9.782379e-05	6.761060e-04	6.684183e-04	4.870932e-04	4.473916e-05	5.490263e-04
		3	1.019845e-05	9.828942e-05	9.832589e-05	6.732373e-04	6.737570e-04	4.834152e-04	4.454944e-05	5.504070e-04
	5	0.8	1.244829e-05	9.531211e-05	9.396941e-05	6.619346e-04	6.416342e-04	4.826693e-04	5.332704e-05	5.291213e-04
		1.2	1.249009e-05	9.521021e-05	9.430075e-05	6.591944e-04	6.455362e-04	4.793880e-04	5.328594e-05	5.298140e-04
		3	1.243846e-05	9.494079e-05	9.433740e-05	6.558832e-04	6.470110e-04	4.762892e-04	5.291300e-05	5.292168e-04
100	0.5	0.8	1.077757e-05	2.777679e-05	5.325025e-05	3.578425e-04	1.996079e-04	1.399423e-04	2.449027e-05	1.6587403e-05
		1.2	1.079650e-05	2.783058e-05	5.345474e-05	3.575318e-04	2.009702e-04	1.396488e-04	2.453675e-05	1.6648210e-05
		3	1.080659e-05	2.781308e-05	5.353955e-05	3.569310e-04	2.016452e-04	1.393644e-04	2.456291e-05	1.6823475e-05
	1.5	0.8	5.150707e-06	5.123168e-05	9.715353e-05	6.805084e-04	3.468376e-04	2.562672e-04	2.258768e-05	2.7598640e-05
		1.2	5.122698e-06	5.131691e-05	9.782379e-05	6.761060e-04	3.498661e-04	2.546388e-04	2.244171e-05	2.8562473e-05
		3	5.095712e-06	5.155513e-05	9.832589e-05	6.732373e-04	3.527910e-04	2.534140e-04	2.230138e-05	2.8435698e-05
	5	0.8	6.385606e-06	4.890564e-05	9.396941e-05	6.619346e-04	3.332358e-04	2.505722e-04	2.688259e-05	2.6485231e-05
		1.2	6.397454e-06	4.886504e-05	9.430075e-05	6.591944e-04	3.345725e-04	2.494731e-04	2.687361e-05	2.6987453e-05
		3	6.340974e-06	4.880075e-05	9.433740e-05	6.558832e-04	3.350901e-04	2.481000e-04	2.663499e-05	2.6578941e-05

From the table-3, it is observed that the Bayes risk of survival function is minimum under the SELF, QLF, SLLF when β<1 and α<1than β<1 and α>1, β>1 and α<1 than β>1 and α>1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function is maximum under the GELF, WLF when β<1 and α>1 than β<1 and α<1, β>1 and α<1 than β>1 and α>1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function for Weibull distribution using non-informative prior (Extension of Jeffrey's) under SELF is better than using other loss functions proposed in this study.

The Bayes risk of survival function for two parameter Weibull distribution using informative prior (Lognormal-Inverted Gamma) under various loss functions is obtained and presented in Table-4.

From the table-4, it is observed that the Bayes risk of survival function is minimum under the SELF, WLF when β<1 and α>1, β>1 and α>1 than β<1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function is maximum under the GELF, QLF, SLLF when β<1 and α>1 than β<1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function for Weibull distribution using non-informative prior (Lognormal-Inverted gamma) under SELF is better than using other loss functions proposed in this study.

ISSN 2348-1218 (print)
International Journal of Interdisciplinary Research and Innovations ISSN 2348-1226 (online)
Vol. 6, Issue 2, pp: (339-353), Month: April - June 2018, Available at: www.researchpublish.com

Table-4: Estimation of Bayes risk of Survival Function under Lognormal-Inverted Gamma prior with hyperparameters a=0.4, b=0.6 and c=0.9.

N	β	α	$R(\hat{S}(t_i))_{BS}$	$R(\hat{S}(t_i))_{BG}$ k=0.6	k=-0.6	k=1.6	k=-1.6	$R(\hat{S}(t_i))_{BQ}$	$R(\hat{S}(t_i))_{BW}$	$R(\hat{S}(t_i))_{BSL}$
25	0.5	0.8	4.247933e-05	1.042675e-04	1.032485e-04	7.259696e-04	7.110812e-04	5.340861e-04	1.003680e-04	0.579389e-04
		1.2	4.242947e-05	1.045762e-04	1.032988e-04	7.291869e-04	7.106086e-04	5.368080e-04	1.000195e-04	0.580366e-04
		3	4.219367e-05	1.048524e-04	1.029726e-04	7.337281e-04	7.064748e-04	5.410068e-04	9.907469e-05	0.580115e-04
	1.5	0.8	2.111536e-05	1.912253e-04	1.916436e-04	1.284471e-03	1.288378e-03	8.882945e-04	9.102045e-05	1.076693e-03
		1.2	2.099766e-05	1.926466e-04	1.911657e-04	1.300483e-03	1.278769e-03	9.020567e-04	9.119609e-05	1.079338e-03
		3	2.063684e-05	1.951268e-04	1.892871e-04	1.332562e-03	1.252209e-03	9.300142e-04	9.120207e-05	1.080726e-03
	5	0.8	2.587927e-05	1.867347e-04	1.876853e-04	1.249465e-03	1.260175e-03	8.619481e-04	1.0843378e-04	1.053616e-03
		1.2	2.582640e-05	1.878323e-04	1.874636e-04	1.260414e-03	1.254846e-03	8.70365-e-04	1.0866435e-04	1.056436e-03
		3	2.581295e-05	1.897237e-04	1.866053e-04	1.284600e-03	1.242181e-03	8.914073e-04	1.0937378e-04	1.058592e-03
50	0.5	0.8	2.163816e-05	1.032482e-04	5.338378e-05	3.748984e-04	3.713476e-04	2.799799e-04	5.018555e-05	0.298318e-04
		1.2	2.160960e-05	1.032990e-04	5.344022e-05	3.766316e-04	3.712782e-04	2.815217e-04	5.006574e-05	0.298959e-04
		3	2.151326e-05	1.029723e-04	5.323841e-05	3.783477e-04	3.689510e-04	2.835194e-04	4.975986e-05	0.298532e-04
	1.5	0.8	1.032973e-05	9.79257e-04	9.798936e-05	6.718430e-04	6.724426e-04	4.835472e-04	4.488039e-05	0.548167e-04
		1.2	1.024258e-05	9.833945e-04	9.786497e-05	6.76076e1-04	6.687039e-04	4.871230e-04	4.470409e-05	0.549159e-04
		3	1.016133e-05	9.888921e-04	9.722271e-05	6.846225e-04	6.597744e-04	4.959646e-04	4.471787e-05	0.548889e-04
	5	0.8	1.263053e-05	9.480087e-04	9.501011e-05	6.516442e-04	6.547116e-04	4.711981e-04	5.301132e-05	0.530698e-04
		1.2	1.261331e-05	9.494254e-04	9.486057e-05	6.538124e-04	6.524624e-04	4.734869e-04	5.309366e-05	0.530676e-04
		3	1.249098e-05	9.502661e-04	9.429631e-05	6.571540e-04	6.459860e-04	4.778277e-04	5.291545e-05	0.625075e-04
100	0.5	0.8	1.084852e-05	2.843710e-04	2.836312e-05	1.977431e-04	1.964181e-04	1.469635e-04	2.500330e-05	0.158613e-04
		1.2	1.083692e-05	2.859237e-04	2.844636e-05	1.988954e-04	1.965856e-04	1.477443e-04	2.496297e-05	0.159331e-04
		3	1.081237e-05	2.867041e-04	2.835949e-05	2.000356e-04	1.954876e-04	1.488520e-04	2.488150e-05	0.159284e-04
	1.5	0.8	5.167376e-06	5.106136e-04	5.110710e-05	3.497672e-04	3.501210e-04	2.532998e-04	2.254044e-05	0.286334e-04
		1.2	5.128875e-06	5.135180e-04	5.117656e-05	3.521648e-04	3.495102e-04	2.549434e-04	2.243660e-05	0.287392e-04
		3	5.086607e-06	5.186991e-04	5.119034e-05	3.566320e-04	3.471666e-04	2.582050e-04	2.234664e-05	0.289100e-04
	5	0.8	6.430705e-06	4.871654e-04	4.880297e-05	3.363746e-04	3.375929e-04	2.465949e-04	2.680192e-05	0.272532e-04
		1.2	6.427581e-06	4.877129e-04	4.875354e-05	3.371642e-04	3.368368e-04	2.474186e-04	2.682543e-05	0.272546e-04
		3	6.352897e-06	4.884330e-04	4.858936e-05	3.384573e-04	3.346036e-04	2.486442e-04	2.663894e-05	0.272309e-04

The Bayes risk of survival function for two parameter Weibull distribution using informative prior (Lognormal-Inverted Gamma) under various loss functions is obtained and presented in Table-5.

Table-5: Estimation of Bayes risk of Survival Function under Lognormal-Inverted Gamma prior with hyperparameters a=1.4, b=1.6 and c=1.9.

N	B	α	$R(\hat{S}(t_i))_{BS}$	$R(\hat{S}(t_i))_{BG}$ k=0.6	k=-0.6	k=1.6	k=-1.6	$R(\hat{S}(t_i))_{BQ}$	$R(\hat{S}(t_i))_{BW}$	$R(\hat{S}(t_i))_{BSL}$
25	0.5	0.8	4.196950e-05	1.038268e-04	1.007805e-04	7.345662e-04	6.879657e-04	5.480479e-04	1.001110e-04	5.706600e-04
		1.2	4.191776e-05	1.040534e-04	1.009498e-04	7.364438e-04	6.889940e-04	5.495518e-04	9.993228e-05	5.717524e-04
		3	4.192357e-05	1.044322e-04	1.011640e-04	7.397769e-04	6.899067e-04	5.520824e-04	9.981088e-05	5.733752e-04
	1.5	0.8	2.113981e-05	1.910415e-04	1.917815e-04	1.282400e-03	1.290271e-03	8.864081e-04	9.076616e-05	1.076543e-03
		1.2	2.108012e-05	1.917994e-04	1.921209e-04	1.288847e-03	1.291059e-03	8.912825e-04	9.070151e-05	1.079644e-03
		3	2.090690e-05	1.934575e-04	1.928421e-04	1.301800e-03	1.291446e-03	8.998539e-04	9.040182e-05	1.086654e-03
	5	0.8	2.596976e-05	1.851437e-04	1.884482e-04	1.232276e-03	1.274303e-03	8.477712e-04	1.072560e-04	1.051264e-03
		1.2	2.597215e-05	1.857873e-04	1.888360e-04	1.236909e-03	1.275628e-03	8.504684e-04	1.072524e-04	1.054246e-03
		3	2.612199e-05	1.870006e-04	1.894054e-04	1.248294e-03	1.278980e-03	8.586500e-04	1.075561e-04	1.058797e-03
50	0.5	0.8	2.154530e-05	5.362554e-05	5.285758e-05	3.777743e-04	3.655841e-04	2.841635e-04	5.018703e-05	2.968184e-04
		1.2	2.152370e-05	5.376973e-05	5.297288e-05	3.789043e-04	3.662638e-04	2.850569e-04	5.012105e-05	2.975324e-04
		3	2.150943e-05	5.385310e-05	5.299823e-05	3.797964e-04	3.662528e-04	2.858935e-04	5.006004e-05	2.978316e-04
	1.5	0.8	1.034034e-05	9.780633e-05	9.806300e-05	6.704542e-04	6.736129e-04	4.823624e-04	4.476899e-05	5.480276e-04
		1.2	1.026840e-05	9.802196e-05	9.819176e-05	6.715791e-04	6.733002e-04	4.827635e-04	4.452456e-05	5.491859e-04
		3	1.023157e-05	9.834900e-05	9.826637e-05	6.745287e-04	6.724693e-04	4.852304e-04	4.445932e-05	5.504038e-04
	5	0.8	1.265423e-05	9.437656e-05	9.522200e-05	6.464070e-04	6.588905e-04	4.661744e-04	5.266800e-05	5.300611e-04
		1.2	1.265042e-05	9.441478e-05	9.519579e-05	6.469113e-04	6.584118e-04	4.666819e-04	5.269224e-05	5.300975e-04
		3	1.256520e-05	9.432199e-05	9.497951e-05	6.467955e-04	6.562848e-04	4.672086e-04	5.242245e-05	5.292732e-04
100	0.5	0.8	1.081626e-05	2.845111e-05	2.814041e-05	1.989678e-04	1.939620e-04	1.486837e-04	2.499217e-05	1.580123e-04
		1.2	1.080657e-05	2.858280e-05	2.825371e-05	1.997786e-04	1.945237e-04	1.491260e-04	2.496528e-05	1.587531e-04
		3	1.080263e-05	2.863582e-05	2.826880e-05	1.997786e-04	1.945139e-04	1.495532e-04	2.494547e-05	1.589320e-04
	1.5	0.8	5.169896e-06	5.101974e-05	5.114017e-05	3.492023e-04	3.506403e-04	2.527535e-04	2.251495e-05	2.863086e-04
		1.2	5.134674e-06	5.122974e-05	5.130675e-05	3.492023e-04	3.512942e-04	2.532432e-04	2.239424e-05	2.874157e-04
		3	5.101703e-06	5.162125e-05	5.159537e-05	3.504434e-04	3.520198e-04	2.541092e-04	2.228409e-05	2.895817e-04
	5	0.8	6.436048e-06	4.857966e-05	4.888080e-05	3.525995e-04	3.390722e-04	2.447802e-04	2.671343e-05	2.723596e-04
		1.2	6.436390e-06	4.859612e-05	4.887427e-05	3.345782e-04	3.389322e-04	2.661598e-05	2.449820e-04	2.723873e-04
		3	6.370900e-06	4.859608e-05	4.883152e-05	3.347842e-04	3.382688e-04	2.672222e-05	2.448054e-04	2.722963e-04

From the table-5, it is observed that the Bayes risk of survival function is minimum under the SELF, WLF when β<1 and α>1, β>1 and α>1 than β<1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function is maximum under the GELF, QLF, SLLF when β<1 and α>1 than β<1 and α<1 for n=25. In the same behavior of the values of scale and shape parameters of the Bayes risk of survival function is reported when n=50 and n=100. The Bayes risk of survival function for Weibull distribution using non-informative prior (Lognormal-Inverted Gamma) under SELF is better than using other loss functions proposed in this study.

6. CONCLUSION

In this study, we obtained the Bayes risk of survival function of Weibull distribution using non-informative and informative priors such as Jeffrey's, Extension of Jeffrey's and Lognormal-Inverted gamma priors under Squared error loss function, General entropy loss function, Quadratic loss function, Weighted loss function, Squared logarithmic loss function by applying Lindley's approximation rule and illustrate the methodology through simulation technique. By comparing the estimated values of Bayes risk of survival function of Weibull distribution using various loss functions, the risk assuming under SELF is the least one among the cases studied. It is found that when the sample size as well as iteration process is increased the Bayes risk of survival function is decreased. Finally, among all the cases the two parameter Weibull model with Lognormal-inverted gamma prior under Squared error loss function is performed well in this study.

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